Journal of the Nigerian Association of Mathematical Physics
Volume 49, (Jan., 2019 Issue), pp 111 – 116

Linear Model for Fuel Optimizing Speed on roads Connecting Lafia Metropolis to Neighbouring communities

Olagunju A. S.[1], Akeyede I.[1], Ayoo P. V[1]., and Habila A. A.[2]

[1]Department of Mathematics, Federal University Lafia, 950101, Nigeria
[2]Department of computer Science, Federal University Lafia, 950101, Nigeria

Abstract

Fueling cost represents a very significant portion of vehicle maintenance cost. For some vehicle it takes up to 50% of the total maintenance cost. Fuel consumption has been established to depend largely on factors like vehicle characteristics, highway characteristics and the attitudes of drivers. In this study, a real time fuel consumption model was developed for the 5 major roads connecting neighboring communities to Lafia metropolis. In most of these communities over speeding in a bid to minimize fuel consumption is paramount and this study was designed to experimentally arrive at fuel minimizing speed for these roads. To develop the model, a large speed fuel experiment involving more than 1000km of driving was carried out for a speed range of 40km/h to 120km/h using a Toyota corolla car with fittings to take instantaneous data of fuel consumption and road characteristics. A model was constructed for fuel consumption as a function of speed and number of applied brake. The full speed equation for each of these roads was minimized to yield a range of numerical values for optimal fuel for each road. These equations are found to be consistent and within the neighbourhood of EPA (Environmental Protection Agency) minimal fuel speed of the used vehicle.

Mathematics subject classification: 11C08, 11D04

Keywords: Optimal fuel consumption; road characteristic; vehicle characteristics.

Corresponding Author: Olagunju A. S., Email: olagunjuas@gmail.com

1. Introduction

The research presented here takes place on 5 major roads connecting neighbouring communities to Lafia, the state capital of Nasarawa state, Nigeria. The state is located in the North-central geopolitical zone of Nigeria, known for its agrarian population. Most farming activities go on in the neighbouring communities.

According to statistics obtained from 2016 Federal Road Safety Corps 2016 report [1], road crash is on the increase and the major cause is over speeding, as a result of this, there is a need for empirical evidence to validate or otherwise, the need for too much speed on these roads. The experimental component of this research work took place under a free flow driving session that involves driving to and fro on each of these roads for a speed range of 40km/hr, 60km/hr, 80km/hr 100km/hr and 120km/hr. The research is aimed at arriving at speed that minimizes fuel consumption, to achieve this; we develop a simple model that is able to estimate fuel consumption on the basis of variables that are easily accessible via the use of specialized devices and mobile applications (fuel consumption meter, gradient and curvature meter).

This report focuses on the development, testing and the use of this model to determine the minimal fuel consumption as a function of speed and number of times of applied brake. The principal experiment concerned driving sessions that took place on these roads which involved gathering data for road characteristics (rise or fall, curvature, bumps, potholes) and the vehicles fuel consumption. These data were used for the estimation of parameters for the model.

The paper is organized as follows: **Section 1** Contains introduction; **Section 2** Reviews of relevant literatures on past fuel – speed experiment; in **Section 3** Experimental data are presented, tools for data collection are also presented. In **Section 4** Analysis of data is carried out and models are presented. Results are presented and discussed in **section 5** while conclusions are also drawn alongside.

2. Problem Formulation

A lot of measures have been adopted to maximise fuel economy of vehicles. These include improvements in vehicle technologies (Catalytic converter, alternative fuels, and engine efficiency), highway designs, traffic regulation etc. Another effective and promising technique as illustrated in [2] is the monitoring and control of drivers' behaviour - eco-driving which is defined as a decision-making process aimed at influencing fuel economy and emissions intensity of a vehicle so as to reduce it environmental impact [3].

The upward surge and unstable fuel price has made fuel minimizing strategies more necessary especially in developing economies. In nooks and crannies of northern Nigeria where this research took place, over speeding to maximize fuel economy is paramount and as a result, this research is aimed at creating speed-fuel optimising model for the 5 roads that connect neighbouring communities to Lafia. There are several variables that affect fuel consumption; these include vehicle characteristics, road characteristics (Paved or unpaved, rise and fall, curvature and etc) and driving attitudes [4-6]. To keep the model as simple as possible a level of abstraction was applied such that fuel consumption is estimated on the basis of 2 variables – speed and number of applied brake. Consumption estimated with this model reflects the average consumption for a certain class of vehicles (1.6 litre engine with curb weight that ranges between 1,280 kg and 1330 kg).

Some works on fuel-speed model adopt microscopic approach, where developed models are classified according to different criteria. First of which is the fuel supply – gasoline, diesel and hybrid vehicles. The second is the basis of the models, some of which have a basic principle of the traction law [1]. These models estimate fuel consumption as a function of the mass of the vehicle, the aerodynamic drag coefficient, the vehicle frontal area, the acceleration and speed, the road (gradient and curvature) [7-8]. With the use of carbon balance method in some other experiments, fuel consumption has been indirectly inferred from gas emissions. Other cases such as NEDC (New European Driving cycle) and the US FTP (Federal Test Procedure) models are based on laboratory-test performed on a chassis dynamometer, that implements standard driving cycles [9-10]. In some other researches, a nonlinear regression model was developed based on a polynomial combination of instantaneous speed and acceleration, one of such model used data collected at the Oak Ridge National Laboratory using chassis dynamometer [2].

In this study, we propose a simple polynomial model developed for each road under consideration, the model uses as input variables, the vehicle instantaneous speed, the characteristic of the road which leads to application of brake. The intention is for the estimation of fuel consumption and to provide empirical evidences of fuel-minimising speed while driving on these roads. The mind-set behind the construction of this model is to have a model that is as simple as possible in order to be easily adapted on different platforms for real time use. It is equally based on variables that are easily obtained with simple devices.

3. Data Source

The principal experiment was carried out on the five major roads that connect neighbouring communities to Lafia metropolis, these roads includes; Lafia – Makurdi (88km), Lafia – Awe (80km), Lafia – Doma (16km), Lafia – Asakio (40km), Lafia – Akwanga (60km). The study involves driving to and fro for the range of the considered speed – 40km/hr, 60km/hr, 80km/hr, 100km/hr and 120km/hr. These roads are paved but some obstructions to speed (pots holes and speed-bumps) exist on few portions of the roads. The reduction in speed to climb the speed-bumps or manoeuvre the pot-holes was insignificant at low speed e. g. 40km/hr (less than 10km/hr lower than the stipulated speed) but at high speed, these produced a high level of significance as brake was applied several times.

The speed-fuel experiment was carried out under free-flow scenario where the roads were free of traffic. The experiment was conducted with the vehicle driven within the stipulated speed and to move freely according to the nature of the road for example brake was applied to climb speed bumps and manoeuvre the rough sections of the road. The vehicle in use is a petrol-engine Toyota Corolla (2014 model, 1.6 litres engine) that uses indirect fuel injection. The car was equipped with fuel flow sensor fitted at the entry point of injector, also fuel flow display meter was connected to display data from fuel flow sensor. Gradient meter that measures rise and fall and curvature-meter that measure the degree of curvature were also fitted to give real time data.

Fuel consumptions at intervals were recorded, number of significant brake application was taken into account, rise and fall were also taken into consideration as well as curvature. Since these experiments are specifically tied to these roads under consideration, only speed and number of brake times was used as variables, the reason for this is that other factors (curvature, speed-bumps and etc) all leads to application of brake. Also this equally helps to keep the model simple.

Fuel consumption is usually represented in 2 variables namely: the instantaneous fuel consumption (FC_{inst}), which expresses the fuel consumption for every second, and the litre per kilometre fuel consumption (FC_{km}), which expresses the fuel consumption in one kilometre for a uniform speed. The following formula gives FC_{inst} from fuel metering [mg i]:

$$FC_{inst}\left[\frac{l}{s}\right] = \frac{4 \times RPM \times fuel\ metering}{2 \times 1000 \times 60 \times 770} \quad (1)$$

Where:

4 is the number of cylinders in the engine;
RPM (Revolution per minute) is rated for 2 because there is injection each in 2 RPM;
1000 is used to switch from mg to g;
770 is the density of petrol fuel expressed g/l.

The FC_{km} is computed using the formula:

$$FC_{km}\left[\frac{l}{km}\right] = \frac{FC_{inst}}{speed \times 3600} \quad (2)$$

where speed is expressed in km/h.

It should be noted that fuel consumption display (FCD) gauge expresses average fuel usage over a distance; the instant fuel consumption calculates a vehicle's fuel consumption instantly. As the vehicle moves, sensors continuously detect fuel flow rate and the fuel flow display unit displays them to the driver.

Given in Table 1 is the data gathered from these experiments and used for analysis in this study, this refers to subset of the collected variables.

Table 1 **TABLE OF SPEED-FUEL AND NUMBER OF BRAKING TIMES**

SPEED	**40km/hr**		**60km/hr**		**80km/hr**		**100km/hr**		**120km/hr**	
ROADS	Fuel used	Brakes applied	Fuel used	Brakes applied	Fuel used	Brakes applied	Fuel used	Brakes applied	Fuel used	Brakes applied
Lafia – Makurdi	7.625	20	7.050	25	5.400	35	7.800	52	8.900	57
Lafia – Awe	5.550	8	4.900	12	4.190	14	6.850	20	7.025	22
Lafia – Doma	0.850	0	1.150	1	0.700	2	2.250	5	2.40	5
Lafia – Assakio	4.850	42	3.100	49	3.565	62	3.850	64	3.925	66
Lafia – Akwanga	5.650	9	3.200	30	3.260	35	4.025	44	4.450	49

Table 2 **TABLE OF KILOMETRE PER LITRE AND BRAKE PER KILOMETRE**

Speed	**Variables**	**Lafia – Makurdi**	**Lafia – Awe**	**Lafia – Doma**	**Lafia – Assakio**	**Lafia – Akwanga**
40km/hr	KPL	11.5410	14.4144	18.8235	7.7419	8.8496
	BPK	0.2273	0.1	0	0.625	0.18
60km/hr	KPL	12.4823	16.3265	13.913	13.3333	15.625
	BPK	0.2841	0.15	0.0625	0.8333	0.6
80km/hr	KPL	16.2963	19.0931	22.8571	10.7143	15.3374
	BPK	0.3977	0.175	0.125	1.375	0.7
100km/hr	KPL	11.2821	11.6788	7.1111	9.4118	12.4224
	BPK	0.5909	0.25	0.3125	1.4583	0.88
120km/hr	KPL	9.8880	11.3879	6.6667	10.7865	11.236
	BPK	0.6477	0.275	0.3125	1.5417	0.98

Table 3 **TABLE OF LITRE PER 100 KILOMETRES**

Speed	**Lafia – Makurdi**	**Lafia – Awe**	**Lafia – Doma**	**Lafia – Assakio**	**Lafia – Akwanga**
40km/hr	8.6648	6.9375	5.3125	12.9167	11.3
60km/hr	8.0114	6.125	7.1875	7.5000	6.4
80km/hr	6.1364	5.2375	4.375	9.3333	6.52
100km/hr	8.8636	8.5625	14.0625	10.6250	8.05
120km/hr	10.1136	8.7813	15	9.2708	8.9

4. Results

Fuel-consumption models developed for these roads are based only on speed (v) and number of significant application of brake. This is a deliberate choice as we are interested in adapting variables that can be easily collected. Also there exists a direct correlation between some road characteristics (e. g. curvature, roughness, rise and fall) and number of applied brake.

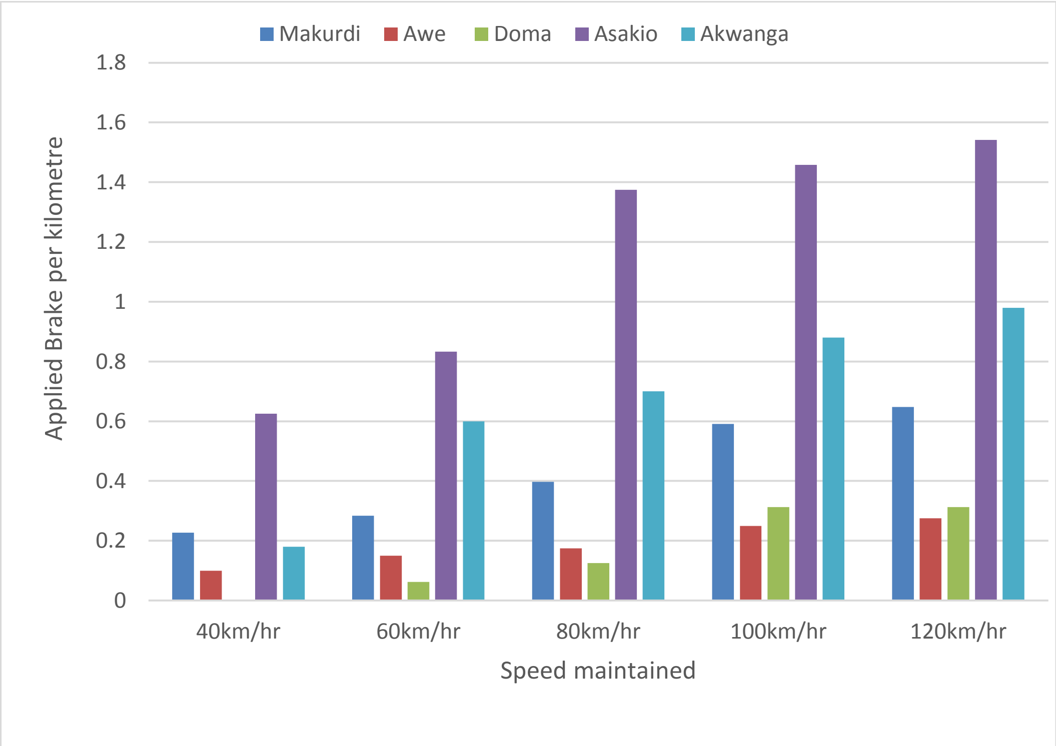

Figure 1 Pictorial display of average number of brakes applied per kilometre *versus* speed. The correlation between these two variables is positive for the entire speed range.

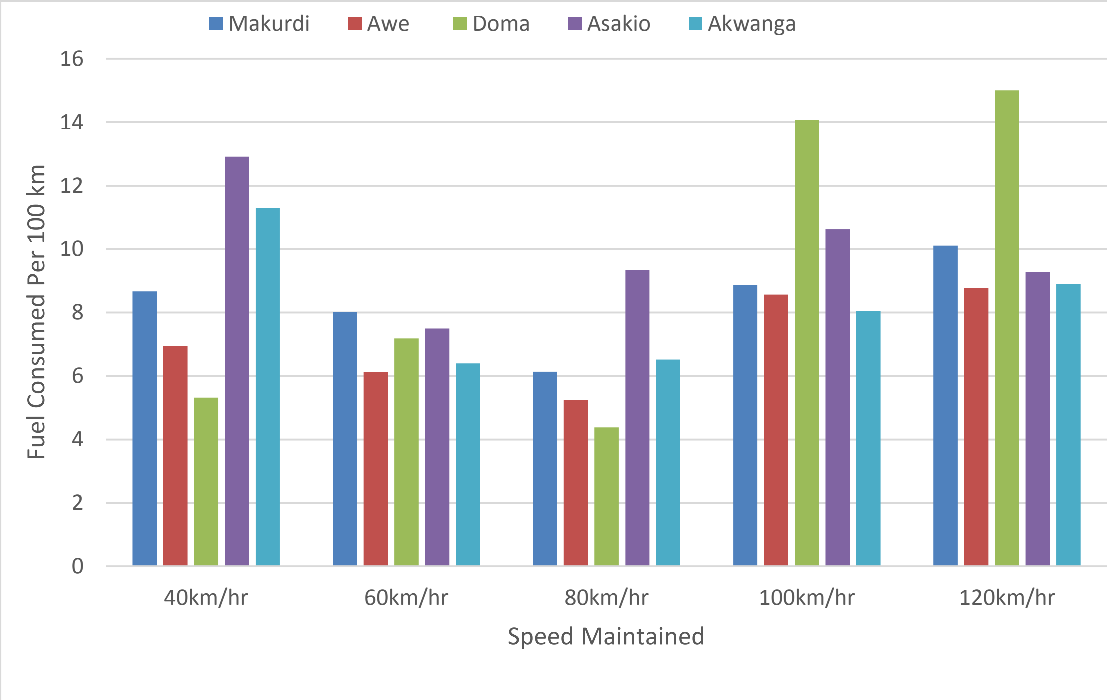

Figure 2 This clustered column gives the pictorial relationship that was obtained from the experiment; this corresponds with figures obtained from fuel economy database for Toyota Corolla model.

4.1 Model specification and estimation of parameters

Identification of the fuel-consumption model was based on data from the experiment. The model variables were arrived at by using fitting algorithm implemented in the MATLAB toolbox. As discussed in [2], this method easily applies to linear models and can be very effective to be adopted for real-time identification procedures. The linear model is calibrated as follows:

$$FC = \beta_0 + \beta_1 v + \beta_2 x + \beta_3 v^2 + \beta_4 xv \qquad (3)$$

Where FC is the amount of fuel consumed in litres, v is the speed and x represents the number of applied brake. The parameters β_i were estimated using MATLAB toolbox and the results are reported for each road as follows:

Table 4 **Table of coefficients for equation (5)**

Roads		Constants	v	v^2	x	Vx	R- square	SSE
Lafia – Makurdi	β_i	17.22	0.2605	-0.005175	-1.186	0.015	1	$2.299e^{-27}$
Lafia – Awe		10.1	-0.02592	-0.00185	-0.71	0.01602	1	$9.309\ e^{-29}$
Lafia – Doma		-34	1.256	-0.009625	-16	0.1737	1	$7.571\ e^{-28}$
Lafia – Assakio		9.628	-0.00504	-0.001687	-0.5991	0.008933	1	$4.203\ e^{-28}$
Lafia – Akwanga		8.607	0.02269	- 0.001516	-0.3925	0.005813	1	$5.838\ e^{-29}$

4.2 Fuel optimizing speed

As earlier stated, the aim of this work is to develop a model that can be used to arrive at fuel minimizing speed on these roads. To achieve this, we minimize equation (3) with respect to *v*. i. e. we determined *v* in terms of *x* by setting:

$$\frac{\partial FC}{\partial v} = 0 \tag{4}$$

The value of *x* at the point where the minimum fuel was achieved from the experiment is hereby substituted into (4) to arrive at the fuel minimizing speed.

For each of the roads we have these as follow:

Table 5 **Table of fuel-optimizing speed from equation (4)**

Roads	v at $\frac{\partial FC}{\partial v} = 0$	Fuel optimizing speed at x where FC is minimal
Lafia – Makurdi	$v = 1.4493x + 25.1691$	75.9 km/hr
Lafia – Awe	$v = 4.3297x - 7.0054$	70.9 km/hr
Lafia – Doma	$v = 9.0234x + 65.2468$	83.3 km/hr
Lafia – Asakio	$v = 2.6476x - 1.4938$	59.4 km/hr
Lafia – Akwanga	$v = 1.9172x + 7.4835$	74.6 km/hr

5. Discussion

Fuel minimizing speed has been arrived at for the 5 roads under consideration. The Lafia – Assakio road which happen to be the worst of the roads in terms of roughness especially at Assakio end of the road has the lowest fuel optimizing speed (59.4 km/hr). During data analysis, the very rough portion of the road where the stipulated speed could not be attained was removed from the entire dataset. We therefore make use of 24km in place of 40 kilometres as the highest speed that was attained in the experiment at rough section was 60 km/hr. The road also has the highest number of speed bumps per kilometre. It was discovered that every form of interruption to smooth ride on the road leads to a very significant increase in fuel consumption, hence it is recommended that drivers that ply this road should drive in anticipation of the speed bumps and the bad portion of the road, so as to maintain speed that is not so high thereby able to achieve short braking distance to climb speed bumps or navigate through the rough sections. This road also accounts for the lowest kilometre covered per litre at 40 km/hr.

Lafia – Akwanga road records the highest degree of rise and fall, number of curvatures on this road is also the highest. This road accounts for the highest kilometre per litre at 60km/h and the lowest at 40km/h. On the average, fuel optimizing speed for this road is 74.6km/hr. However, this speed could hardly be maintained at extreme curve.

6. Conclusion

Reduction in fuel consumption is a possibility that can be achieved by adopting more efficient driving styles. Our research addresses the myth of using high speed to reduce fuel consumption. The accuracy of our model was verified by the data from EPA estimated fuel economy for Toyota corolla. From the findings of this research, it is discovered that despite the relative simplicity of the model’s structure and few input parameters, the proposed model provides a good estimation of minimal fuel speed. It is worth noting that driver’s attitude affects consumption greatly, apart from maintaining the afore-discussed fuel-minimizing speed, it is advisable the drivers attain these speeds gradually by pressing the throttle pedal steadily. Also driving should be done to minimize application of brake, on time and gradual application of brake is better and more fuel-economical than sudden braking of speed. Tyres should be inflated according to the manufacturer’s standard pressure.

We recommend that further research effort be extended to other categories of vehicles (Trucks, bus, tricycle and etc) as many of these are equally in large use in Nasarawa state where this research was conducted.

Acknowledgement

This research was sponsored by Tertiary Education Trust fund (TetFund) under 2017 institutional-based research grant, through Federal University Lafia, Nigeria.

References

[1] 2016 Report of Federal Road safety Corps (FRSC) Annual Report, May **2017,** ISSN: 2467 - 8759

[2] A Linear Model for the Estimation of Fuel Consumption and the Impact Evaluation of Advanced Driving Assistance Systems. Gennaro Nicola B.; Francesco G.; Luigi P.; and Maria Russo S.; *Sustainability* **2015,** 7, 14326-14343;doi:10.3390/su71014326

[3] Carteni, A. Urban Sustainable Mobility. Part 1: Rationality in Transport Planning. Available online: http://www.transportproblems.polsl.pl/pl/Archiwum/2014/zeszyt4/2014t9z4-04.pdf(accesses on 19 October 2015).

[4] Chi, Y.; Guo, Z.; Zheng, Y.; Zhang, X. Scenarios Analysis of the Energies' Consumption and Carbon Emissions in China Based on a Dynamic CGE Model. *Sustainability* **2015,** 7, 14326-14343;doi:10.3390/su71014326.

[5] Sivak, M.; Schoettle, B. Eco-driving: Strategic, tactical, and operational decisions of the driver that influence vehicle fuel economy. *Transp. Policy* **2012,** 22,96-99.

[6] Leung, D.Y.C.; Williams, D.J. Modeling of motor vehicle fuel consumption and emissions using a power-based model. *Environ. Monit. Assess.* **2000,** 65,21-29.

[7] Rakha, H.A.; Ahn, K.; Trani, A. Development of VT-Micro model for estimating hot-stabilized light duty vehicle and truck emissions. *Transp. Res. Part D Transp. Environ,* **2004,** 9,49-74.

[8] Post, K.; Kent, J. H.; Tomlin, J.; Carruthers, N. Fuel consumption and emission modelling by power demand and a comparison with other models. *Transp. Res. Part A General* **1984**, *18*, 191–213.

[9] Akcelik, R. Efficiency and drag in the power-based model of fuel consumption. *Transport. Res. B Methodological* **1989**, *23*, 376–385.

[10] Wong, J.Y. *Theory of Ground Vehicles*; John Wiley & Sons: Hoboken, NJ, USA, 2001.

[11] Rakha, H.A.; Ahn, K.; Moran, K.; Searens, B.; Van den Bulck, E. Virginia Tech Comprehensive Power-Based Fuel Consumption Model: Model development and testing. *Transp. Res. Part D Transp. Environ.* **2011**, *16*, 492–503.